

Paper Reference 9FM0/3A
Pearson Edexcel
Level 3 GCE

Further Mathematics
Advanced
PAPER 3A: Further Pure Mathematics 1

Wednesday 18 June 2025 – Afternoon
Time: 1 hour 30 minutes

Question Paper

THIS DOCUMENT MUST BE RETURNED TO
PEARSON AT THE END OF THE EXAMINATION.

In the boxes below, write your name, centre number and candidate number.

Surname					
Other names					
Centre Number					
Candidate Number					

YOU MUST HAVE

**Mathematical Formulae and Statistical Tables (Green),
calculator**

YOU WILL BE GIVEN

Diagram Booklet

Answer Booklet

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet or in the separate Diagram Booklet – there may be more space than you need.

You should show sufficient working to make your methods clear.

Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

There are 9 questions in this Question Paper.

The total mark for this paper is 75

The marks for EACH question are shown in brackets

– use this as a guide as to how much time to spend on each question.

There may be spare copies of some diagrams.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

Turn over

1. (a) Refer to the table for Question 1 in the Diagram Booklet.

Given that

$$y = e^{\operatorname{cosec}^2 x}$$

complete the table in the Diagram Booklet with the value of y corresponding to $x = 2$, giving your answer to 2 decimal places.

(1 mark)

- (b) Use Simpson's rule, with all the values of y in the completed table, to estimate, to one decimal place, the value of

$$\int_{1.5}^{2.5} e^{\operatorname{cosec}^2 x} dx$$

(3 marks)

(continued on the next page)

1. continued.

(c) Using your answer to part (b) and making your method clear, estimate the value of

$$\int_{1.5}^{2.5} e^{\cot^2 x} dx$$

(2 marks)

(Total for Question 1 is 6 marks)

2. Refer to the information for Question 2 in the Diagram Booklet.

It shows the Taylor series expansion of $f(x)$ about $x = a$

Given that

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 2xy = 4 \quad (\text{I})$$

(a) show that

$$\frac{d^4y}{dx^4} = a\frac{dy}{dx} + bx\frac{d^2y}{dx^2} + c\frac{d^3y}{dx^3}$$

where a , b and c are integers to be determined.
(4 marks)

(continued on the next page)

2. continued.

Hence, given that

$$y = 1 \text{ and } \frac{dy}{dx} = 1 \text{ when } x = 2$$

- (b) determine a Taylor series solution, in ascending powers of $(x - 2)$, up to and including the term in $(x - 2)^4$, of the differential equation **(I)**, giving each coefficient in simplest form.
(4 marks)

(Total for Question 2 is 8 marks)

3. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Use algebra to determine the values of x for which

$$\frac{x^2 - 4}{|x - 5|} > 4x + 8$$

(6 marks)

(Total for Question 3 is 6 marks)

4. $y = \sqrt{(15 + e^{2x})}$

(a) Show that

$$\frac{d^2y}{dx^2} = \frac{e^{2x}(e^{2x} + 30)}{(15 + e^{2x})^{\frac{3}{2}}}$$

(3 marks)

(b) Hence determine the Maclaurin series expansion for y , in ascending powers of x , up to and including the term in x^2 , giving each term in simplest form.

(2 marks)

(c) Use the series expansion for $\sin x$ in ascending powers of x to determine, in simplest form, the first 2 non-zero terms in ascending powers of x of the series for $\sin 3x$

(1 mark)

(continued on the next page)

Turn over

4. continued.

(d) Use the answers to parts (b) and (c) to show that

$$\lim_{x \rightarrow 0} \frac{\sqrt{(15 + e^{2x})} - 4}{\sin 3x} = \frac{1}{12}$$

(2 marks)

(Total for Question 4 is 8 marks)

5. (a) Use the substitution

$$t = \tan\left(\frac{x}{2}\right) \text{ to show that}$$

$$\int \frac{1}{1 + \operatorname{cosec} x} dx = \int \frac{4t}{(1 + t^2)(1 + t)^2} dt$$

(3 marks)

(b) Hence show that

$$\int \frac{1}{1 + \operatorname{cosec} x} dx = Ax + \frac{B}{1 + \tan\left(\frac{x}{2}\right)} + k$$

where **A** and **B** are constants to be determined
and **k** is an arbitrary constant.

(5 marks)

(Total for Question 5 is 8 marks)

6. (i) Use L'Hospital's Rule to show that

$$\lim_{x \rightarrow 0} \frac{1 - \cos 7x}{x \sin 9x} = \frac{49}{18}$$

(4 marks)

(ii) $y = x^2 e^{3x}$

- (a) Use Leibnitz's theorem to show that,
for $k \in \mathbb{N}$

$$\frac{d^k y}{dx^k} = 3^{k-2} e^{3x} (Ax^2 + Bkx + Ck(k-1))$$

where **A**, **B** and **C** are integers to be determined.

(5 marks)

(continued on the next page)

6. (ii) continued.

(b) Hence determine the values of x for which

$$\frac{d^9 y}{dx^9} = 0$$

(2 marks)

(Total for Question 6 is 11 marks)

7. The concentration, $P \text{ mg m}^{-3}$, of a pollutant in a reservoir, t days after the pollutant entered the reservoir, is modelled by the differential equation

$$\frac{1}{P} \frac{dP}{dt} = 4tP^2 - 1 \quad (\text{I})$$

- (a) Show that the transformation

$$x = \frac{1}{P^2} \text{ transforms equation (I) into the equation}$$

$$\frac{dx}{dt} - 2x = -8t \quad (\text{II})$$

(3 marks)

(continued on the next page)

7. continued.

Given that

$$P = 0.5 \text{ when } t = 0$$

(b) solve differential equation **(II)** to show that, according to the model,

$$P^2 = \frac{1}{4t + 2 + ke^{2t}}$$

where **k** is a constant to be determined.

(6 marks)

Given that the concentration of the pollutant in the reservoir, 3 days after the pollutant entered the reservoir, was 0.034 mg m^{-3}

(c) comment on the reliability of the model, giving a reason for your answer.

(2 marks)

(Total for Question 7 is 11 marks)

8. The ellipse **E** has equation

$$\frac{x^2}{64} + \frac{y^2}{36} = 1$$

The line **L** has equation

y = mx + c where **m** and **c** are constants.

(a) Show that the **x** coordinates of the points of intersection of **E** and **L** satisfy the equation

$$(16m^2 + 9)x^2 + 32cmx + 16c^2 + k = 0$$

where **k** is a constant to be determined.

(2 marks)

Hence, given that the line **L**

- is a tangent to **E**
- passes through the point **(10, 20)**

(b) determine the possible equations of **L**

(5 marks)

(Total for Question 8 is 7 marks)

9. The parallelogram **P** has vertices **E**, **F**, **G** and **H**, with coordinates

$$\mathbf{E}(10, -1, -6)$$

$$\mathbf{F}(7, -2, 7)$$

$$\mathbf{G}(10, 2, 9)$$

$$\mathbf{H}(13, 3, -4)$$

- (a) Determine the exact area of **P**, giving your answer as a simplified surd.
(3 marks)

The line **L** passes through **E** and is perpendicular to the plane containing **P**

- (b) Determine an equation for **L** giving your answer in the form

$$(\underline{\mathbf{r}} - \underline{\mathbf{a}}) \times \underline{\mathbf{b}} = \underline{\mathbf{0}}$$

where $\underline{\mathbf{a}}$ and $\underline{\mathbf{b}}$ are constant vectors.

(2 marks)

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Turn over

9. continued.

Given that

- P is one face of a parallelepiped
- the opposite face of the parallelepiped lies in the plane Π
- the point $(25, -4, 13)$ lies in Π

(c) determine the volume of the parallelepiped.
(5 marks)

(Total for Question 9 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER